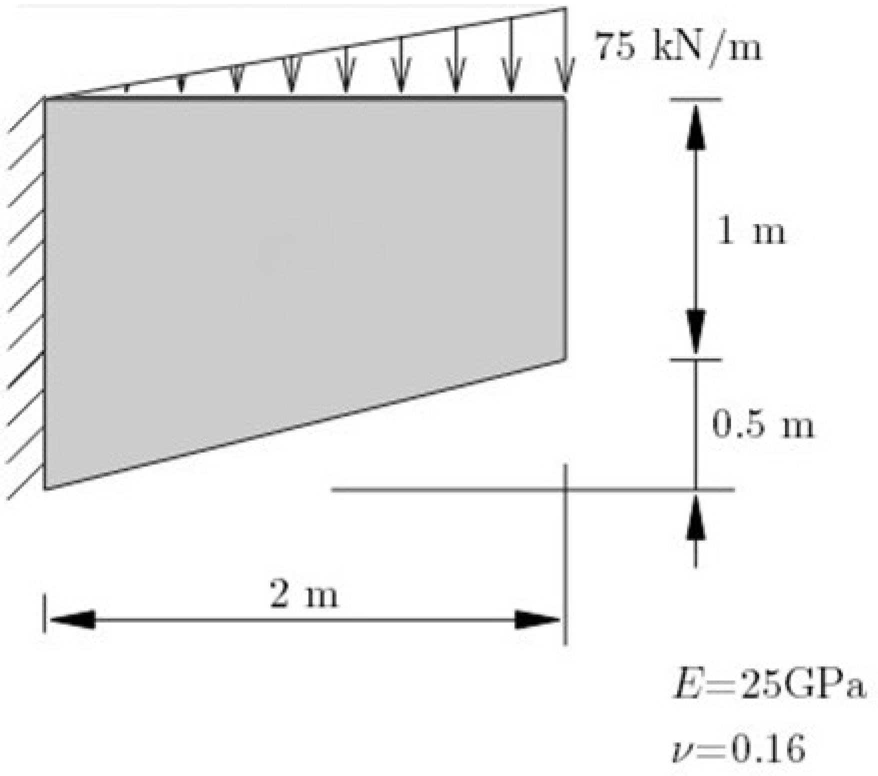

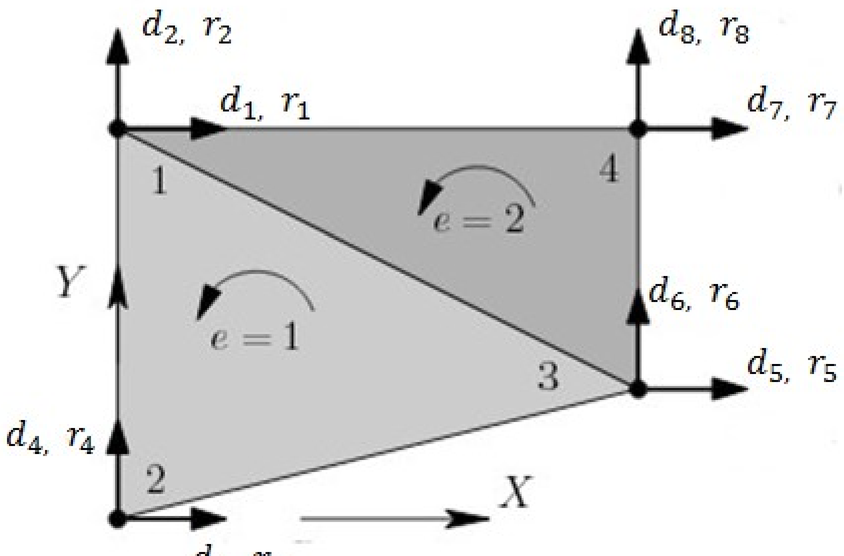

$ORIGIN := 1$

Stałe materiałowe

Wzór na obliczenie pola elementów

$E := 25e6 \qquad v := 0.16$

$$wsp := \begin{pmatrix} 0 & 1.5 \\ 0 & 0 \\ 2 & 0.5 \\ 2 & 1.5 \end{pmatrix} \qquad top := \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \end{pmatrix} \qquad A(e) := \frac{1}{2} \cdot \left\| \begin{bmatrix} wsp_{(top_{e,1}),1} & wsp_{(top_{e,2}),1} & wsp_{(top_{e,3}),1} \\ wsp_{(top_{e,1}),2} & wsp_{(top_{e,2}),2} & wsp_{(top_{e,3}),2} \\ 1 & 1 & 1 \end{bmatrix} \right\|$$

Obliczenie modułu sprężystości

$$D := \frac{E}{(1+v)\cdot(1-2\cdot v)} \cdot \begin{pmatrix} 1-v & v & 0 \\ v & 1-v & 0 \\ 0 & 0 & \frac{1-2\cdot v}{2} \end{pmatrix} \qquad D = \begin{pmatrix} 2.662\times 10^{7} & 5.071\times 10^{6} & 0 \\ 5.071\times 10^{6} & 2.662\times 10^{7} & 0 \\ 0 & 0 & 1.078\times 10^{7} \end{pmatrix}$$

Funkcje kształtu

$$Nd1(x,y,e) := \left[wsp_{(top_{e,2}),2} - wsp_{(top_{e,3}),2}\right]\cdot\frac{x}{2\cdot A(e)} + \left[wsp_{(top_{e,3}),1} - wsp_{(top_{e,2}),1}\right]\cdot\frac{y}{2\cdot A(e)}$$

$$N1(x,y,e) := Nd1(x,y,e) + \frac{\left[wsp_{(top_{e,2}),1}\cdot wsp_{(top_{e,3}),2} - wsp_{(top_{e,3}),1}\cdot wsp_{(top_{e,2}),2}\right]}{2\cdot A(e)}$$

$$Nd2(x,y,e) := \left[wsp_{(top_{e,3}),2} - wsp_{(top_{e,1}),2}\right]\cdot\frac{x}{2\cdot A(e)} + \left[wsp_{(top_{e,1}),1} - wsp_{(top_{e,3}),1}\right]\cdot\frac{y}{2\cdot A(e)}$$

$$N2(x,y,e) := Nd2(x,y,e) + \frac{\left[wsp_{(top_{e,3}),1}\cdot wsp_{(top_{e,1}),2} - wsp_{(top_{e,1}),1}\cdot wsp_{(top_{e,3}),2}\right]}{2\cdot A(e)}$$

$$Nd3(x,y,e) := \left[wsp_{(top_{e,1}),2} - wsp_{(top_{e,2}),2}\right]\cdot\frac{x}{2\cdot A(e)} + \left[wsp_{(top_{e,2}),1} - wsp_{(top_{e,1}),1}\right]\cdot\frac{y}{2\cdot A(e)}$$

$$N3(x,y,e) := Nd3(x,y,e) + \frac{\left[wsp_{(top_{e,1}),1}\cdot wsp_{(top_{e,2}),2} - wsp_{(top_{e,2}),1}\cdot wsp_{(top_{e,1}),2}\right]}{2\cdot A(e)}$$

element 1

$$N1(x,y,1) \rightarrow -0.16666666666666666667\cdot x + 0.66666666666666666667\cdot y$$

$$N2(x,y,1) \rightarrow -0.33333333333333333333\cdot x + -0.66666666666666666667\cdot y + 1.0$$

$$N3(x,y,1) \rightarrow 0.5\cdot x$$

element 2

$$N1(x,y,2) \rightarrow -0.5\cdot x + 1.0$$

$$N2(x,y,2) \rightarrow -1.0\cdot y + 1.5$$

$$N3(x,y,2) \rightarrow 0.5\cdot x + 1.0\cdot y - 1.5$$

Macierz funkcji kształtu

$$N(x,y,e) := \begin{pmatrix} N1(x,y,e) & 0 & N2(x,y,e) & 0 & N3(x,y,e) & 0 \\ 0 & N1(x,y,e) & 0 & N2(x,y,e) & 0 & N3(x,y,e) \end{pmatrix}$$

Macierz pochodnych funkcji kształtu

$$B(x,y,e) := \begin{pmatrix} \frac{d}{dx}N1(x,y,e) & 0 & \frac{d}{dx}N2(x,y,e) & 0 & \frac{d}{dx}N3(x,y,e) & 0 \\ 0 & \frac{d}{dy}N1(x,y,e) & 0 & \frac{d}{dy}N2(x,y,e) & 0 & \frac{d}{dy}N3(x,y,e) \\ \frac{d}{dy}N1(x,y,e) & \frac{d}{dx}N1(x,y,e) & \frac{d}{dy}N2(x,y,e) & \frac{d}{dx}N2(x,y,e) & \frac{d}{dy}N3(x,y,e) & \frac{d}{dx}N3(x,y,e) \end{pmatrix}$$

$$A(1) = 1.5 \qquad A(2) = 1$$

$$B(0,0,1) = \begin{pmatrix} -0.167 & 0 & -0.333 & 0 & 0.5 & 0 \\ 0 & 0.667 & 0 & -0.667 & 0 & 0 \\ 0.667 & -0.167 & -0.667 & -0.333 & 0 & 0.5 \end{pmatrix} \qquad B(0,0,2) = \begin{pmatrix} -0.5 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & -0.5 & -1 & 0 & 1 & 0.5 \end{pmatrix}$$

Macierze sztywności

$$Ke(e) := B(0,0,e)^{T}\cdot D\cdot B(0,0,e)\cdot A(e)$$

$$Ke(1) = \begin{pmatrix} 8.293\times10^{6} & -2.641\times10^{6} & -4.965\times10^{6} & -2.747\times10^{6} & -3.328\times10^{6} & 5.388\times10^{6} \\ -2.641\times10^{6} & 1.82\times10^{7} & 1.056\times10^{5} & -1.685\times10^{7} & 2.535\times10^{6} & -1.347\times10^{6} \\ -4.965\times10^{6} & 1.056\times10^{5} & 1.162\times10^{7} & 5.282\times10^{6} & -6.656\times10^{6} & -5.388\times10^{6} \\ -2.747\times10^{6} & -1.685\times10^{7} & 5.282\times10^{6} & 1.954\times10^{7} & -2.535\times10^{6} & -2.694\times10^{6} \\ -3.328\times10^{6} & 2.535\times10^{6} & -6.656\times10^{6} & -2.535\times10^{6} & 9.984\times10^{6} & 0 \\ 5.388\times10^{6} & -1.347\times10^{6} & -5.388\times10^{6} & -2.694\times10^{6} & 0 & 4.041\times10^{6} \end{pmatrix}$$

$$Ke(2) = \begin{pmatrix} 6.656\times10^{6} & 0 & 0 & 2.535\times10^{6} & -6.656\times10^{6} & -2.535\times10^{6} \\ 0 & 2.694\times10^{6} & 5.388\times10^{6} & 0 & -5.388\times10^{6} & -2.694\times10^{6} \\ 0 & 5.388\times10^{6} & 1.078\times10^{7} & 0 & -1.078\times10^{7} & -5.388\times10^{6} \\ 2.535\times10^{6} & 0 & 0 & 2.662\times10^{7} & -2.535\times10^{6} & -2.662\times10^{7} \\ -6.656\times10^{6} & -5.388\times10^{6} & -1.078\times10^{7} & -2.535\times10^{6} & 1.743\times10^{7} & 7.923\times10^{6} \\ -2.535\times10^{6} & -2.694\times10^{6} & -5.388\times10^{6} & -2.662\times10^{7} & 7.923\times10^{6} & 2.932\times10^{7} \end{pmatrix}$$

Macierze Boole'a

$$Bo(el) := \left| \begin{array}{l} Bo_{6,8} \leftarrow 0 \\ \text{for } i \in 1..2 \\ \quad \left| \begin{array}{l} Bo_{i,\,2\cdot(top_{el,1}-1)+i} \leftarrow 1 \\ Bo_{i+2,\,2\cdot(top_{el,2}-1)+i} \leftarrow 1 \\ Bo_{i+4,\,2\cdot(top_{el,3}-1)+i} \leftarrow 1 \end{array} \right. \end{array} \right.$$

$$Bo(1) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \qquad Bo(2) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Agregacja macierzy sztywności

$$K := Bo(1)^{T}\cdot Ke(1)\cdot Bo(1) + Bo(2)^{T}\cdot Ke(2)\cdot Bo(2)$$

$$K = \begin{pmatrix} 1.495\times10^{7} & -2.641\times10^{6} & -4.965\times10^{6} & -2.747\times10^{6} & -3.328\times10^{6} & 7.923\times10^{6} & -6.656\times10^{6} & -2.535\times10^{6} \\ -2.641\times10^{6} & 2.089\times10^{7} & 1.056\times10^{5} & -1.685\times10^{7} & 7.923\times10^{6} & -1.347\times10^{6} & -5.388\times10^{6} & -2.694\times10^{6} \\ -4.965\times10^{6} & 1.056\times10^{5} & 1.162\times10^{7} & 5.282\times10^{6} & -6.656\times10^{6} & -5.388\times10^{6} & 0 & 0 \\ -2.747\times10^{6} & -1.685\times10^{7} & 5.282\times10^{6} & 1.954\times10^{7} & -2.535\times10^{6} & -2.694\times10^{6} & 0 & 0 \\ -3.328\times10^{6} & 7.923\times10^{6} & -6.656\times10^{6} & -2.535\times10^{6} & 2.076\times10^{7} & 0 & -1.078\times10^{7} & -5.388\times10^{6} \\ 7.923\times10^{6} & -1.347\times10^{6} & -5.388\times10^{6} & -2.694\times10^{6} & 0 & 3.066\times10^{7} & -2.535\times10^{6} & -2.662\times10^{7} \\ -6.656\times10^{6} & -5.388\times10^{6} & 0 & 0 & -1.078\times10^{7} & -2.535\times10^{6} & 1.743\times10^{7} & 7.923\times10^{6} \\ -2.535\times10^{6} & -2.694\times10^{6} & 0 & 0 & -5.388\times10^{6} & -2.662\times10^{7} & 7.923\times10^{6} & 2.932\times10^{7} \end{pmatrix}$$

Wektor prawej strony - równoważniki obciążenia

funkcja obciążenia

$$f(p1_x, p2_x, p1_y, p2_y, L, s) := \begin{bmatrix} p1_x\cdot\left(1-\frac{s}{L}\right)+p2_x\cdot\frac{s}{L} \\ p1_y\cdot\left(1-\frac{s}{L}\right)+p2_y\cdot\frac{s}{L}\end{bmatrix} \qquad F(s) := f(0,0,0,-75,2,s) \qquad F(s) \rightarrow \begin{pmatrix} 0 \\ -\frac{75\cdot s}{2}\end{pmatrix}$$

Zastępnik dla elementu 2 - obciążenie wzdłuż osi x=s i dla y=1.5

$$zt2(s) := N(s, 1.5, 2)^T\cdot F(s)$$

$$i := 1..6$$

$$z2_i := \int_0^2 zt2(s)_i\, ds \qquad z2 = \begin{pmatrix} 0 \\ -25 \\ 0 \\ 0 \\ 0 \\ -50 \end{pmatrix} \qquad p := Bo(2)^T\cdot z2 \qquad p = \begin{pmatrix} 0 \\ -25 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -50 \end{pmatrix}$$

Warunki brzegowe - zablokowane nr stopni swobody

$$war := \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$

Uwzględnienie warunków brzegowych

$$i := 1..4 \quad I := identity(8) \quad Id_{8,8} := 0 \quad Id_{war_i, war_i} := 1 \quad Ip := I - Id \quad KK := Ip\cdot K\cdot Ip + Id \quad pp := Ip\cdot p$$

Rozwiązanie równania MES

$$d := KK^{-1}\cdot pp$$

$$r := K\cdot d - p$$

$$d = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1.585\times 10^{-6} \\ -1.042\times 10^{-5} \\ 3.092\times 10^{-6} \\ -1.229\times 10^{-5} \end{pmatrix} \qquad r = \begin{pmatrix} -66.667 \\ 42.923 \\ 66.667 \\ 32.077 \\ 0 \\ 0 \\ 0 \\ -4.974\times 10^{-14} \end{pmatrix}$$

Powrót do elementów

$$d1 := Bo(1)\cdot d \qquad \varepsilon_1 := B(0,0,1)\cdot d1 \qquad d2 := Bo(2)\cdot d \qquad \varepsilon_2 := B(0,0,2)\cdot d2$$

$$d1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1.585\times 10^{-6} \\ -1.042\times 10^{-5} \end{pmatrix} \qquad \varepsilon_1 = \begin{pmatrix} -7.925\times 10^{-7} \\ 0 \\ -5.208\times 10^{-6} \end{pmatrix} \qquad d2 = \begin{pmatrix} 0 \\ 0 \\ -1.585\times 10^{-6} \\ -1.042\times 10^{-5} \\ 3.092\times 10^{-6} \\ -1.229\times 10^{-5} \end{pmatrix} \qquad \varepsilon_2 = \begin{pmatrix} 1.546\times 10^{-6} \\ -1.875\times 10^{-6} \\ -1.468\times 10^{-6} \end{pmatrix}$$

$$\sigma_1 := D\cdot\varepsilon_1 \qquad \sigma_2 := D\cdot\varepsilon_2$$

$$\sigma_1 = \begin{pmatrix} -21.099 \\ -4.019 \\ -56.117 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 31.648 \\ -42.088 \\ -15.824 \end{pmatrix}$$

$$\sigma_{1z} := v\cdot\left(\sigma_{1_1} + \sigma_{1_2}\right) \qquad \sigma_{1z} = -4.019 \qquad \sigma_{2z} := v\cdot\left(\sigma_{2_1} + \sigma_{2_2}\right) \qquad \sigma_{2z} = -1.67$$

Wartość przemieszczeń w środku ES

e - nr elementu
j - nr współrzędnej

$$X_s(e,j) := \frac{\sum_{i=1}^{3} wsp_{top_{e,i},j}}{3}$$

$X_s(1,1) = 0.667$

$X_s(1,2) = 0.667$

$X_s(2,1) = 1.333$

$X_s(2,2) = 1.167$

$$u1 := N\left(X_s(1,1), X_s(1,2), 1\right) \cdot d1$$

$$u2 := N\left(X_s(2,1), X_s(2,2), 2\right) \cdot d2$$

$$u1 = \begin{pmatrix} -5.283 \times 10^{-7} \\ -3.472 \times 10^{-6} \end{pmatrix}$$

$$u2 = \begin{pmatrix} 5.023 \times 10^{-7} \\ -7.569 \times 10^{-6} \end{pmatrix}$$