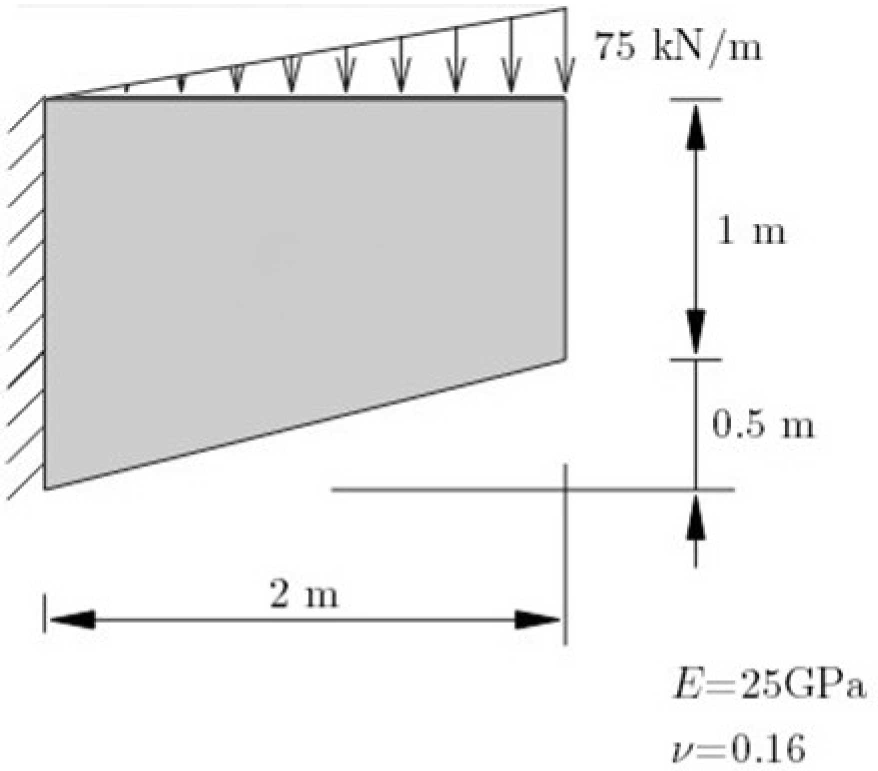

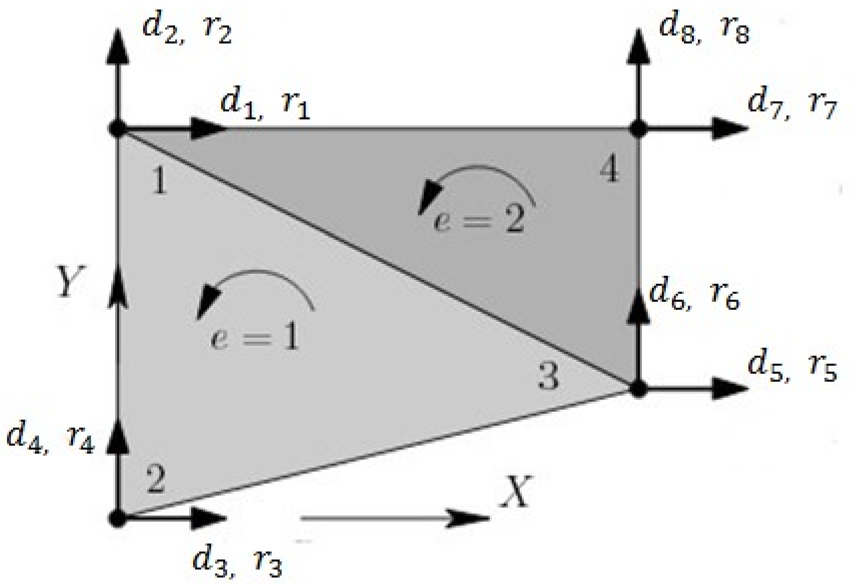

$ORIGIN := 1$

Stałe materiałowe

Wzór na obliczenie pola elementów

$E := 25e6 \qquad v := 0.16$

$$wsp := \begin{pmatrix} 0 & 1.5 \\ 0 & 0 \\ 2 & 0.5 \\ 2 & 1.5 \end{pmatrix} \qquad top := \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \end{pmatrix} \qquad A(e) := \frac{1}{2} \cdot \left\| \begin{vmatrix} wsp_{\left(top_{e,1}\right),1} & wsp_{\left(top_{e,2}\right),1} & wsp_{\left(top_{e,3}\right),1} \\ wsp_{\left(top_{e,1}\right),2} & wsp_{\left(top_{e,2}\right),2} & wsp_{\left(top_{e,3}\right),2} \\ 1 & 1 & 1 \end{vmatrix} \right\|$$

Obliczenie modułu sprężystości

$$D := \frac{E}{(1+v)\cdot(1-2\cdot v)} \cdot \begin{pmatrix} 1-v & v & 0 \\ v & 1-v & 0 \\ 0 & 0 & \frac{1-2\cdot v}{2} \end{pmatrix} \qquad D = \begin{pmatrix} 2.662\times 10^{7} & 5.071\times 10^{6} & 0 \\ 5.071\times 10^{6} & 2.662\times 10^{7} & 0 \\ 0 & 0 & 1.078\times 10^{7} \end{pmatrix}$$

Macierz pochodnych funkcji kształtu

$$b(e,i,j,k) := wsp_{(top_{e,i}),k} - wsp_{(top_{e,j}),k}$$

$$B(e) := \frac{1}{2\cdot A(e)}\cdot\begin{pmatrix} b(e,2,3,2) & 0 & b(e,3,1,2) & 0 & b(e,1,2,2) & 0 \\ 0 & b(e,3,2,1) & 0 & b(e,1,3,1) & 0 & b(e,2,1,1) \\ b(e,3,2,1) & b(e,2,3,2) & b(e,1,3,1) & b(e,3,1,2) & b(e,2,1,1) & b(e,1,2,2) \end{pmatrix}$$

$A(1) = 1.5$ $A(2) = 1$

$$B(1) = \begin{pmatrix} -0.167 & 0 & -0.333 & 0 & 0.5 & 0 \\ 0 & 0.667 & 0 & -0.667 & 0 & 0 \\ 0.667 & -0.167 & -0.667 & -0.333 & 0 & 0.5 \end{pmatrix} \qquad B(2) = \begin{pmatrix} -0.5 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & -0.5 & -1 & 0 & 1 & 0.5 \end{pmatrix}$$

Macierze sztywności

$$Ke(e) := B(e)^T\cdot D\cdot B(e)\cdot A(e)$$

$$Ke(1) = \begin{pmatrix} 8.293\times10^6 & -2.641\times10^6 & -4.965\times10^6 & -2.747\times10^6 & -3.328\times10^6 & 5.388\times10^6 \\ -2.641\times10^6 & 1.82\times10^7 & 1.056\times10^5 & -1.685\times10^7 & 2.535\times10^6 & -1.347\times10^6 \\ -4.965\times10^6 & 1.056\times10^5 & 1.162\times10^7 & 5.282\times10^6 & -6.656\times10^6 & -5.388\times10^6 \\ -2.747\times10^6 & -1.685\times10^7 & 5.282\times10^6 & 1.954\times10^7 & -2.535\times10^6 & -2.694\times10^6 \\ -3.328\times10^6 & 2.535\times10^6 & -6.656\times10^6 & -2.535\times10^6 & 9.984\times10^6 & 0 \\ 5.388\times10^6 & -1.347\times10^6 & -5.388\times10^6 & -2.694\times10^6 & 0 & 4.041\times10^6 \end{pmatrix}$$

$$Ke(2) = \begin{pmatrix} 6.656\times10^6 & 0 & 0 & 2.535\times10^6 & -6.656\times10^6 & -2.535\times10^6 \\ 0 & 2.694\times10^6 & 5.388\times10^6 & 0 & -5.388\times10^6 & -2.694\times10^6 \\ 0 & 5.388\times10^6 & 1.078\times10^7 & 0 & -1.078\times10^7 & -5.388\times10^6 \\ 2.535\times10^6 & 0 & 0 & 2.662\times10^7 & -2.535\times10^6 & -2.662\times10^7 \\ -6.656\times10^6 & -5.388\times10^6 & -1.078\times10^7 & -2.535\times10^6 & 1.743\times10^7 & 7.923\times10^6 \\ -2.535\times10^6 & -2.694\times10^6 & -5.388\times10^6 & -2.662\times10^7 & 7.923\times10^6 & 2.932\times10^7 \end{pmatrix}$$

Macierze Boole'a

$i := 1..2$

$B1_{6,8} := 0$ $B2_{6,8} := 0$

$B1_{i,2\cdot(top_{1,1}-1)+i} := 1$ $B2_{i,2\cdot(top_{2,1}-1)+i} := 1$

$B1_{i+2,2\cdot(top_{1,2}-1)+i} := 1$ $B2_{i+2,2\cdot(top_{2,2}-1)+i} := 1$

$B1_{i+4,2\cdot(top_{1,3}-1)+i} := 1$ $B2_{i+4,2\cdot(top_{2,3}-1)+i} := 1$

$$B1 = \begin{pmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \end{pmatrix} \qquad B2 = \begin{pmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

Agregacja macierzy sztywności

$$K := B1^T \cdot Ke(1) \cdot B1 + B2^T \cdot Ke(2) \cdot B2$$

$$K = \begin{pmatrix} 1.495\times10^{7} & -2.641\times10^{6} & -4.965\times10^{6} & -2.747\times10^{6} & -3.328\times10^{6} & 7.923\times10^{6} & -6.656\times10^{6} & -2.535\times10^{6} \\ -2.641\times10^{6} & 2.089\times10^{7} & 1.056\times10^{5} & -1.685\times10^{7} & 7.923\times10^{6} & -1.347\times10^{6} & -5.388\times10^{6} & -2.694\times10^{6} \\ -4.965\times10^{6} & 1.056\times10^{5} & 1.162\times10^{7} & 5.282\times10^{6} & -6.656\times10^{6} & -5.388\times10^{6} & 0 & 0 \\ -2.747\times10^{6} & -1.685\times10^{7} & 5.282\times10^{6} & 1.954\times10^{7} & -2.535\times10^{6} & -2.694\times10^{6} & 0 & 0 \\ -3.328\times10^{6} & 7.923\times10^{6} & -6.656\times10^{6} & -2.535\times10^{6} & 2.076\times10^{7} & 0 & -1.078\times10^{7} & -5.388\times10^{6} \\ 7.923\times10^{6} & -1.347\times10^{6} & -5.388\times10^{6} & -2.694\times10^{6} & 0 & 3.066\times10^{7} & -2.535\times10^{6} & -2.662\times10^{7} \\ -6.656\times10^{6} & -5.388\times10^{6} & 0 & 0 & -1.078\times10^{7} & -2.535\times10^{6} & 1.743\times10^{7} & 7.923\times10^{6} \\ -2.535\times10^{6} & -2.694\times10^{6} & 0 & 0 & -5.388\times10^{6} & -2.662\times10^{7} & 7.923\times10^{6} & 2.932\times10^{7} \end{pmatrix}$$

Wektor prawej strony - równoważniki obciążenia

sila := (p1 p2 kierunek wez1 wez2)

$$sila := (0\ \ -75\ \ 1\ \ 1\ \ 4)$$

$$p_8 := 0 \qquad l_s := \sqrt{\left[wsp_{(sila_{1,5}),1} - wsp_{(sila_{1,4}),1}\right]^2 + \left[wsp_{(sila_{1,5}),2} - wsp_{(sila_{1,4}),2}\right]^2}$$

$$p_{2\cdot sila_{1,5}-1+sila_{1,3}} := \left(\frac{sila_{1,1}}{6} + \frac{sila_{1,2}}{3}\right)\cdot l_s$$

$$p_{2\cdot sila_{1,4}-1+sila_{1,3}} := \left(\frac{sila_{1,1}}{3} + \frac{sila_{1,2}}{6}\right)\cdot l_s$$

$$p = \begin{pmatrix} 0 \\ -25 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -50 \end{pmatrix}$$

Warunki brzegowe - zablokowane nr stopni swobody

$$war := \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$

Uwzględnienie warunków brzegowych

$$i := 1..4 \quad I := identity(8) \quad Id_{8,8} := 0 \quad Id_{war_i, war_i} := 1 \quad Ip := I - Id \quad KK := Ip\cdot K\cdot Ip + Id \quad pp := Ip\cdot p$$

Rozwiązanie równania MES

$$d := KK^{-1}\cdot pp$$

$$r := K\cdot d - p$$

$$d = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1.585\times10^{-6} \\ -1.042\times10^{-5} \\ 3.092\times10^{-6} \\ -1.229\times10^{-5} \end{pmatrix} \qquad r = \begin{pmatrix} -66.667 \\ 42.923 \\ 66.667 \\ 32.077 \\ 0 \\ 0 \\ 0 \\ 5.684\times10^{-14} \end{pmatrix}$$

Powrót do elementów

$$d1 := B1 \cdot d \qquad d2 := B2 \cdot d$$

$$d1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1.585 \times 10^{-6} \\ -1.042 \times 10^{-5} \end{pmatrix} \qquad d2 = \begin{pmatrix} 0 \\ 0 \\ -1.585 \times 10^{-6} \\ -1.042 \times 10^{-5} \\ 3.092 \times 10^{-6} \\ -1.229 \times 10^{-5} \end{pmatrix}$$

$$\varepsilon_1 := B(1) \cdot d1 \qquad \varepsilon_2 := B(2) \cdot d2$$

$$\varepsilon_1 = \begin{pmatrix} -7.925 \times 10^{-7} \\ 0 \\ -5.208 \times 10^{-6} \end{pmatrix} \qquad \varepsilon_2 = \begin{pmatrix} 1.546 \times 10^{-6} \\ -1.875 \times 10^{-6} \\ -1.468 \times 10^{-6} \end{pmatrix}$$

$$\sigma_1 := D \cdot \varepsilon_1 \qquad \sigma_2 := D \cdot \varepsilon_2$$

$$\sigma_1 = \begin{pmatrix} -21.099 \\ -4.019 \\ -56.117 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 31.648 \\ -42.088 \\ -15.824 \end{pmatrix}$$

$$\sigma_{1z} := v \cdot \left(\sigma_{1_1} + \sigma_{1_2}\right) \qquad \sigma_{1z} = -4.019$$

$$\sigma_{2z} := v \cdot \left(\sigma_{2_1} + \sigma_{2_2}\right) \qquad \sigma_{2z} = -1.67$$