On-line identification of elastic parameters in composite laminates using Lamb waves monitoring and Bayesian filtering

Marek Słoński[1]

[1]*Institute for Computational Civil Engineering, Cracow University of Technology*
Warszawska 24 street, 31-155 Kraków, Poland
e-mail: mslonski@L5.pk.edu.pl

Abstract

This paper presents a new approach to problem of identification of elastic parameters of homogeneous, elastic and hexagonally orthotropic plates. The proposed solution is based on dispersion curves for Lamb waves propagating in free waveguides and Bayesian inference for sequential estimation of elastic parameters. We solve the problem by treating the unknown elastic parameters as state variables of a stationary dynamic system and formulating the sequential identification problem as a Bayesian state estimation problem. We solve the problem by using particle filtering and show results in case of elastic parameters estimation for a thin orthotropic plate.

Keywords: Lamb waves, dispersion curves, thin orthotropic plate, finite element method, Bayesian state estimation, particle filtering

1. Introduction

Currently guided Lamb waves are often used for non-destructive identification of elastic constants of materials. In general, the identification procedures are based on minimization of the discrepancy between experimental and numerical or analytical dispersion curves. Thus, they are unable to characterize reconstruction uncertainty in a systematic manner. In this context Bayesian methods can be useful by offering systematic approach to uncertainty quantification [1]. Bayesian methods are also sequential by solving identification problems recursively. Recently, Słoński in his paper applied particle filter in the problem of identification of elastic parameters of aluminum thin plates [4].

In this work an application of particle filter for sequential stochastic identification of elastic parameters of thin plates using Lamb waves monitoring is proposed. The procedure is based on the comparison of numerical and experimental dispersion curves. The identification results are then presented in the form of a posterior probability density distribution over elastic parameters and the posterior describes the uncertainty. The proposed procedure is verified on an example of pseudo-experimental dispersion curves computed for a thin orthotropic plate.

2. Identification algorithm

We formulate the sequential identification problem as a Bayesian state estimation problem. The elastic parameters are assumed to not change in time, so they are treated as time-independent state variables, see [4] for details. The main goal of Bayesian state estimation is sequential inference of the posterior distribution $p(\mathbf{x}_{k+1}|\mathbf{Y}_{1:k+1})$ starting from a prior distribution $p(\mathbf{x}_k|\mathbf{Y}_{1:k})$. The inference is performed recursively in two steps: prediction step and update (correction) step. In the first step the prediction of state variables distribution $p(\mathbf{x}_{k+1}|\mathbf{y}_k)$ before applying new measurements is done. This distribution is computed using the sum rule of probability and integrating out the state variables as

$$p(\mathbf{x}_{k+1}|\mathbf{Y}_{1:k}) = \int p(\mathbf{x}_{k+1}|\mathbf{x}_k)p(\mathbf{x}_k|\mathbf{Y}_{1:k})d\mathbf{x}_k. \tag{1}$$

Then the new measurements $\mathbf{y}_{k+1}$ are used to update the prior to obtain the posterior distribution $p(\mathbf{x}_{k+1}|\mathbf{Y}_{1:k+1})$ applying the Bayes' rule

$$p(\mathbf{x}_{k+1}|\mathbf{Y}_{1:k+1}) = \frac{p(\mathbf{y}_{k+1}|\mathbf{x}_{k+1})p(\mathbf{x}_{k+1}|\mathbf{Y}_{1:k})}{p(\mathbf{y}_{k+1}|\mathbf{Y}_{1:k})}, \tag{2}$$

where the denominator in (2) is computed from

$$p(\mathbf{y}_{k+1}|\mathbf{Y}_{1:k}) = \int p(\mathbf{y}_{k+1}|\mathbf{x}_{k+1})p(\mathbf{x}_{k+1}|\mathbf{Y}_{1:k})d\mathbf{x}_{k+1}. \tag{3}$$

The update step in Eq. (2) can be also written in the recursive form that is more useful for obtaining particle filter algorithm. Using Bayes' rule we can rewrite Eq. (2) as

$$p(\mathbf{x}_{k+1}|\mathbf{Y}_{1:k+1}) = p(\mathbf{x}_k|\mathbf{Y}_{1:k})\frac{p(\mathbf{y}_{k+1}|\mathbf{x}_{k+1})p(\mathbf{x}_{k+1}|\mathbf{x}_k)}{p(\mathbf{y}_{k+1}|\mathbf{Y}_{1:k})}. \tag{4}$$

The Bayesian state estimation described above gives the posterior distribution over the states. It does not give however, the way to find the solution efficiently using both equations (1) and (2). In addition, the exact inference is intractable and an approximate method has to be applied. In this work a particle filter (PF) algorithm is used. It is based on sequential Monte Carlo sampling and is described for example in [3, 4].

3. Numerical experiments

The effectiveness of the proposed method is assessed by performing numerical exercises for an orthotropic plate. The properties of the plate (Young's moduli, Poisson's ratios and mass density) and the plate thickness, applied in the experiments, are presented in Fig. 1.

Having defined the plate parameters, a pseudo-experimental fundamental antisymmetric dispersion curves A_0 were computed using numerical approach described in [5]. In this approach it is assumed that the dynamic problem is formulated as a plain strain problem and solved by numerical simulations via commercial finite element code Abaqus in a few series of modal analyzes. The numerical model in Abaqus for a 30mm by 1.2mm plate segment has 3600 square CPE4 elements with characteristic length $l_e = 0.1$mm. These dispersion curves were approximated by using basis functions and corresponding parameters that were found with the least squares method. The parameters were treated as the observed variables $\mathbf{y}$. Fig. 1 presents fundamental dispersion curve A0 for the orthotropic plate and its approximation using 5 basis functions proposed in [2].

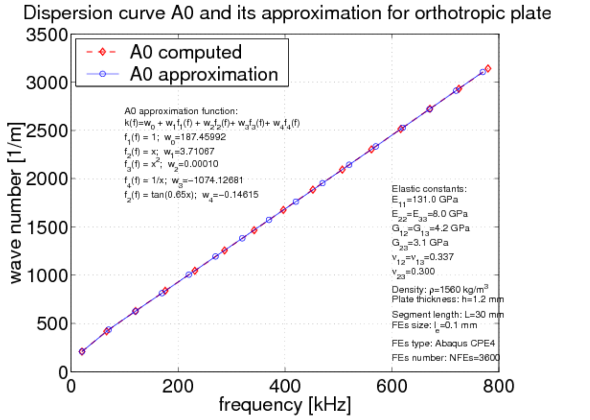

Figure 1: Fundamental dispersion curve for orthotropic plate with thickness h=1.2mm and its approximation using 5 basis functions

Initial and uncertain knowledge about Young's modulus E_1 is represented by a prior distribution $p(x_0)$. We applied a Gaussian prior probability density distribution $p(x_0) = \mathcal{N}(\mu_0, \sigma_0^2)$, with mean value $\mu_0 = 130.0$ GPa and standard deviation σ_0=1.3 GPa (coefficient of variation (CoV) was 1%). Fig. 2 shows the prior.

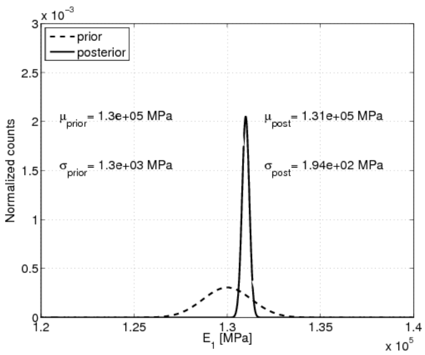

Figure 2: Prior and posterior distributions for Young's modulus

The approximate posterior distribution of Young's modulus given pseudo-experimental dispersion curves $P_N(x_k|\mathbf{y}_k)$ in the k-th step was computed using the particle filter-based identification procedure described above. In experiments, we applied $N = 2000$ particles to obtain the approximate posterior distribution and the number of steps in the sequential identification was set to K=500. The posterior has mean value $\mu_{\text{post}} = 131.0$ GPa and standard deviation σ_{post}=0.19 GPa (CoV is 0.15%).

Fig. 3 shows the sequential nature of the elastic constant identification process by plotting the evolution of the mean value of the posterior distribution and the corresponding plot for the one-standard deviation error bars as a function of the step number. There is also shown a solid horizontal line representing the reference Young's modulus value (131.0 GPa) applied in numerical experiments. From the plot, it may be observed that the estimation process converged to the reference value quite rapidly.

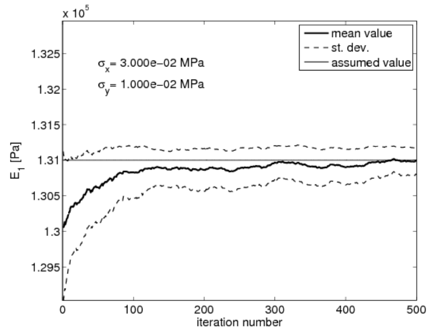

Figure 3: Plot of evolution of mean value of the posterior

Tab. 1 presents statistical parameters of prior and posterior distributions in the form of mean values, standard deviations and coefficients of variation (COV) are given. From the table, it can be stated that the final mean value of the posterior distribution is the same as the reference value. Moreover, the coefficient of variation decreased from 1% for the prior distribution to only 0.15% for the final posterior distribution. Fig. 2 shows the final one-dimensional posterior distribution together with the prior.

Table 1: Statistical parameters of prior and posterior distributions

Parameter	Prior	Posterior
Mean value (GPa)	130.0	131.0
Standard deviation (GPa)	1.3	0.19
COV (%)	1.0	0.15

4. Final conclusions

This paper presents an application of Bayesian methods and particle filter for reconstruction of elastic parameters of plates. The proposed procedure is based on the comparison of experimental and numerical dispersion curves from guided Lamb waves monitoring. Taking into account the assumed experimental errors and considering propagation of errors in the sequential estimation, the uncertainty in the identified value of Young's modulus E_1 is less than 0.5%. More results for other elastic parameters will be presented during the conference.

References

[1] C. Gogu, W. Yin, R. Haftka, P. Ifju, J. Molimard, R. Le Riche, and A. Vautrin. Bayesian identification of elastic constants in multi-directional laminate from Moiré interferometry displacement fields. *Experimental Mechanics*, pages 635–648, 2013.

[2] E. Pabisek, Z. Waszczyszyn, and Ł. Ambroziński. A semi-analytical method for identification of thin elastic plate parameters basing on LWM. *Computer Assisted Meth. Eng. Sci.*, 21(1):5–14, 2014.

[3] S. Russel and P. Norvig. *Artificial Intelligence: A Modern Approach*. Prentice Hall, 3rd Ed., 2010.

[4] M. Słoński. Sequential stochastic identification of elastic constants using Lamb waves and particle filters. *Computer Assisted Meth. Eng. Sci.*, 21(1):15–26, 2014.

[5] Ş. Sorohan, N. Constantin, M. Găvan, and V. Anghel. Extraction of dispersion curves for waves propagating in free complex waveguides by standard finite element codes. *Ultrasonics*, 51(4):503–515, 2011.