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BAYESIAN IDENTIFICATION OF ELASTIC PARAMETERS IN COMPOSITE LAMINATES APPLYING LAMB WAVES MONITORING

Marek Słoński[1]

[1]Cracow University of Technology
Institute for Computational Civil Engineering
ul. Warszawska 24, Kraków, Poland
e-mail: mslonski@L5.pk.edu.pl

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Abstract. *In this paper we consider the problem of identification of elastic parameters of homogeneous, elastic and hexagonally orthotropic plates. The proposed solution of the identification problem is based on dispersion curves for Lamb waves propagating in free waveguides and Bayesian inference for sequential estimation of elastic parameters with uncertainty quantification. In particular we solve the problem by treating the unknown elastic parameters as state variables of a stationary dynamic system and formulating the sequential identification problem as a Bayesian state estimation problem. We solve the problem by using sequential Monte Carlo filter (a.k.a. particle filter). Finally, we present two case studies which correspond either to pseudo-experimental computer simulations or laboratory tests in which the elastic parameters of an aluminum thin plate are estimated. The results confirm that the proposed approach allows to find the unknown elastic parameters and that this approach is also useful for quantification of uncertainty with respect to the elastic parameters.*

1 INTRODUCTION

Currently guided Lamb waves are often used for non-destructive identification of elastic constants of materials. In general, the identification procedure is based on minimization of the discrepancy between experimental and numerical or analytical dispersion curves. Rogers in his highly cited paper presents results of identification of elastic properties of several materials (aluminum, steel, glass) using nonlinear least squares method [5].

Recently, Pabisek et al. in [4] proposed reconstruction of elastic moduli of plates based on fundamental symmetric and antisymmetric dispersion curves obtained through a semi-analytical formulation and corresponding experimental curves. The hybrid method coupled with the neural network based inverse procedure was tested by identification of the elastic properties of a thin aluminum plate.

Most of the guided Lamb wave based identification procedures are deterministic in nature and provide only numerical values of the elastic properties. Thus, they are unable to characterize reconstruction uncertainty in a systematic manner. In this context Bayesian methods can be useful by offering systematic approach to uncertainty quantification. Gogu et al. applied Bayesian methods for identification of elastic constants of an orthotropic plate and they found that Bayesian approach offers more accurate representation of the experimental uncertainty [3].

Bayesian methods are also sequential by solving identification problems recursively. Recently, Słoński in his paper applied particle filter in the problem of identification of elastic parameters of aluminum this plates [7].

In this paper an application of particle filter for sequential stochastic identification of elastic parameters of thin plates using Lamb waves monitoring is proposed. The procedure is based on the comparison of numerical and experimental dispersion curves. The identification results are then presented in the form of a posterior probability density distribution over elastic parameters and the posterior describes the uncertainty. The proposed procedure is verified on an example of pseudo-experimental dispersion curves computed for a thin orthotropic plate.

The paper is organized as follows. Section 2 presents the identification algorithm based on Bayesian state estimation and particle filter for sequential stochastic identification of elastic parameters. Section 3 describes numerical experiments for verification of the proposed procedure together with the results and Section 4 presents the final remarks.

2 IDENTIFICATION ALGORITHM

2.1 Bayesian state estimation problem

We formulate the sequential identification problem as a Bayesian state estimation problem. The elastic parameters are assumed to not change in time, so they are treated as time-independent state variables, see [7] for details. Then the transition equation has the following form:

$$\mathbf{x}_{k+1} = \mathbf{x}_k. \tag{1}$$

The equation (1) is further modified as

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{w}_{k+1}, \tag{2}$$

where $\mathbf{w}_k$ is a noise random variables added for numerical efficiency of particle filter-based identification. In this work, $\mathbf{w}$ is assumed to be a set of independent and identically distributed (iid) Gaussian random variables

$$p(\mathbf{w}) = \mathcal{N}(\mathbf{w}|\mathbf{0}, \boldsymbol{\sigma}_w^2), \tag{3}$$

where $\boldsymbol{\sigma}_w^2$ is a covariance matrix.

The states are recursively estimated using measurements $\mathbf{y}_{k+1}$ that here are defined as the parameters of fundamental antisymmetric dispersion curves A_0. They are related to state variables $\mathbf{x}_{k+1}$ by the nonlinear observation model $\mathbf{h}(\mathbf{x}_{k+1}, \mathbf{v}_{k+1})$ as

$$\mathbf{y}_{k+1} = \mathbf{h}(\mathbf{x}_{k+1}) + \mathbf{v}_{k+1}, \tag{4}$$

where $\mathbf{v}_{k+1}$ is a noise random variables introduced to account for modeling and measurement uncertainties. Here it is also assumed to be a set of independent and identically distributed (iid) Gaussian random variables

$$p(\mathbf{v}) \propto \mathcal{N}(\mathbf{v}|\mathbf{0}, \boldsymbol{\sigma}_v^2), \tag{5}$$

where $\boldsymbol{\sigma}_v^2$ is a covariance function.

The main goal of Bayesian state estimation is sequential inference of the posterior distribution $p(\mathbf{x}_{k+1}|\mathbf{Y}_{1:k+1})$ starting from a prior distribution $p(\mathbf{x}_k|\mathbf{Y}_{1:k})$. The inference is performed recursively in two steps: prediction step and update (correction) step. In the first step the prediction of state variables distribution $p(\mathbf{x}_{k+1}|\mathbf{y}_k)$ before applying new measurements is done. This distribution is computed using the sum rule of probability and integrating out the state variables as

$$p(\mathbf{x}_{k+1}|\mathbf{Y}_{1:k}) = \int p(\mathbf{x}_{k+1}|\mathbf{x}_k) p(\mathbf{x}_k|\mathbf{Y}_{1:k}) d\mathbf{x}_k. \tag{6}$$

Then the new measurements $\mathbf{y}_{k+1}$ are used to update the prior to obtain the posterior distribution $p(\mathbf{x}_{k+1}|\mathbf{Y}_{1:k+1})$ applying the Bayes' rule

$$p(\mathbf{x}_{k+1}|\mathbf{Y}_{1:k+1}) = \frac{p(\mathbf{y}_{k+1}|\mathbf{x}_{k+1}) p(\mathbf{x}_{k+1}|\mathbf{Y}_{1:k})}{p(\mathbf{y}_{k+1}|\mathbf{Y}_{1:k})}, \tag{7}$$

where the denominator in (7) is computed from

$$p(\mathbf{y}_{k+1}|\mathbf{Y}_{1:k}) = \int p(\mathbf{y}_{k+1}|\mathbf{x}_{k+1}) p(\mathbf{x}_{k+1}|\mathbf{Y}_{1:k}) d\mathbf{x}_{k+1}. \tag{8}$$

The update step in Eq. (7) can be also written in the recursive form that is more useful for obtaining particle filter algorithm. Using Bayes' rule we can rewrite Eq. (7) as

$$p(\mathbf{x}_{k+1}|\mathbf{Y}_{1:k+1}) = p(\mathbf{x}_k|\mathbf{Y}_{1:k}) \frac{p(\mathbf{y}_{k+1}|\mathbf{x}_{k+1}) p(\mathbf{x}_{k+1}|\mathbf{x}_k)}{p(\mathbf{y}_{k+1}|\mathbf{Y}_{1:k})}. \tag{9}$$

2.2 Particle filter

The Bayesian state estimation described above gives the posterior distribution over the states. It does not give however, the way to find the solution efficiently using both equations (6) and (7). In addition, the exact inference is intractable and an approximate method has to be applied. In this work a particle filter (PF) algorithm is used. It is based on sequential Monte Carlo sampling and is described below.

In order to implement Bayesian filtering, we approximate the posterior distribution $p(\mathbf{x}_{k+1}|\mathbf{y}_{k+1})$ using N particles $\mathbf{x}_{k+1}^i$, ($i = 1, 2, \ldots, N$), with corresponding importance weights w_{k+1}^i, that is replace the posterior distribution with the empirical distribution

$$P_N(\mathbf{x}_{k+1}) = \sum_{i=1}^{N} w_{k+1}^i \delta(\mathbf{x}_{k+1} - \mathbf{x}_{k+1}^i), \tag{10}$$

where $\delta(\cdot)$ is the Dirac delta function. The weights are computed using sequential importance sampling as

$$w_{k+1}^i = w_k^i \frac{p(\mathbf{y}_{k+1}|\mathbf{x}_{k+1}^i)p(\mathbf{x}_{k+1}^i|\mathbf{x}_k^i)}{\pi(\mathbf{x}_{k+1}^i|\mathbf{x}_k^i, \mathbf{y}_{k+1})}, \tag{11}$$

where $\pi(\mathbf{x}_{k+1}|\mathbf{x}_k, \mathbf{y}_{k+1})$ is the importance distribution such that samples from it can be easily generated.

In general, choosing the optimal importance distribution is rather difficult so for simplicity, the common choice is to apply the transition density as the importance density

$$\pi(\mathbf{x}_{k+1}^i|\mathbf{x}_k^i, \mathbf{y}_{k+1}) = p(\mathbf{x}_{k+1}^i|\mathbf{x}_k^i), \tag{12}$$

that yields a simple equation for computing weights in the next time step as

$$w_{k+1}^i = w_k^i \, p(\mathbf{y}_{k+1}|\mathbf{x}_{k+1}^i). \tag{13}$$

Note that these weights are normalized and satisfy $0 \leq w_k^i \leq 1$ and $\sum_{i=1}^N w_k^i = 1$.

The initial weights are uniform with values $w_1^i = 1/N$ but later during recursive computations they become far from uniform leading to particles degradation (few particles with large weights). As a result the empirical distribution becomes very poor approximation of the state variables distribution $p(\mathbf{x}_{k+1}|\mathbf{Y}_{1:k+1})$. To overcome this particular degradation problem, a sequential resampling procedure is applied. The resampling procedure regenerates the set of particles by replicating the particles with high importance weights and removing samples with low weights.

Finally, the basic particle filter algorithm is as follows. It starts with a population of N initial-state samples, created by sampling from the prior $p(\mathbf{x}_1)$. Then the prediction-update-resample cycle is repeated for each time step [6]:

1. Each sample is propagated forward by sampling the next state value $\mathbf{x}_{k+1}$, given the current value $\mathbf{x}_k$ for the sample, based on the transition model $p(\mathbf{x}_{k+1}|\mathbf{x}_k)$.

2. Each sample is weighted by the likelihood it assigns to the new evidence, $p(\mathbf{y}_{k+1}|\mathbf{x}_{k+1})$.

3. The population is resampled to generate a new population of N samples. Each new sample is selected from the current population; the probability that a particular sample is selected is proportional to its weight. The new samples are unweighted.

A flowchart of the proposed identification algorithm is shown in Fig. 1.

3 NUMERICAL EXPERIMENTS

The effectiveness of the proposed method is assessed by performing numerical exercises for an orthotropic plate. In the experiments, the material properties of the plate (Young's moduli, Poisson's ratios and mass density) and the plate thickness are taken from [1], see Tab. 1 for exact values of these parameters.

Having defined the plate parameters, a pseudo-experimental fundamental antisymmetric dispersion curves A_0 were computed using numerical approach described in [2]. In this approach it is assumed that the dynamic problem is formulated as a plain strain problem and solved by numerical simulations via commercial finite element code Abaqus in a few series of modal analyzes. The numerical model in Abaqus for a 30mm by 1.2mm plate segment has 3600 square

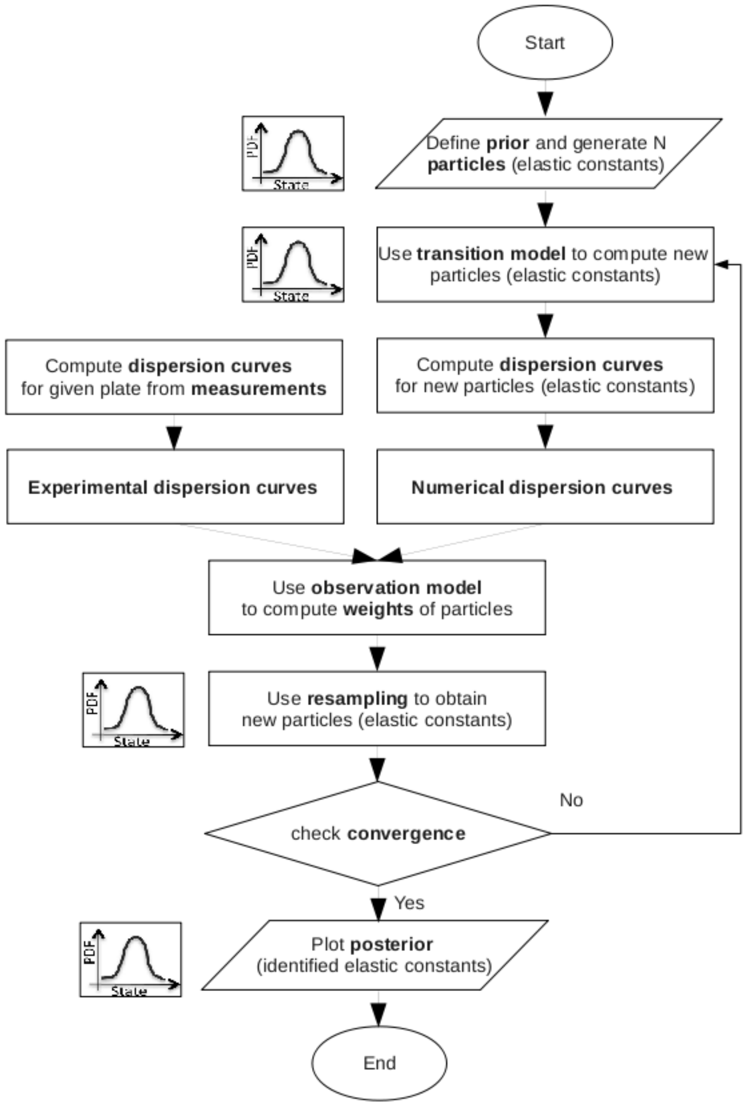

Figure 1: Flowchart of particle filter-based algorithm for elastic parameters identification

CPE4 elements with characteristic length $l_e = 0.1\text{mm}$. These dispersion curves were approximated by using basis functions and corresponding parameters that were found with the least squares method. The parameters were treated as the observed variables $\mathbf{y}$. Fig. 2 presents fundamental dispersion curve A0 for the orthotropic plate with elastic constants defined in Tab. 1 and its approximation using 5 basis functions proposed in [4]:

$$k(f) = \phi_0 w_0 + w_1\phi_1 + w_2\phi_2 + w_3\phi_3 + w_4\phi_4, \tag{14}$$

where k denotes wave number corresponding to frequency f and $\phi_i(f)$ denotes i-th basis function. These basis functions are defined as:

$$\phi_0 = 1,\ \phi_1 = f,\ \phi_2 = f^2,\ \phi_3 = 1/f,\ \phi_4 = \tan(0.65f). \tag{15}$$

3.1 Young's modulus E_1 identification

Initial and uncertain knowledge about Young's modulus E_1 is represented by a prior distribution $p(x_0)$. We applied a Gaussian prior probability density distribution $p(x_0) = \mathcal{N}(\mu_0, \sigma_0^2)$,

Table 1: Assumed values of orthotropic plate parameters applied in numerical experiments

Parameter	E_1(GPa)	E_2(GPa)	ν_{12}(-)	ν_{23}(-)	G_1(GPa)	ρ(kg/m^3)	h(mm)
Assumed value	131	8	0.337	0.3	4.23	1560	1.2

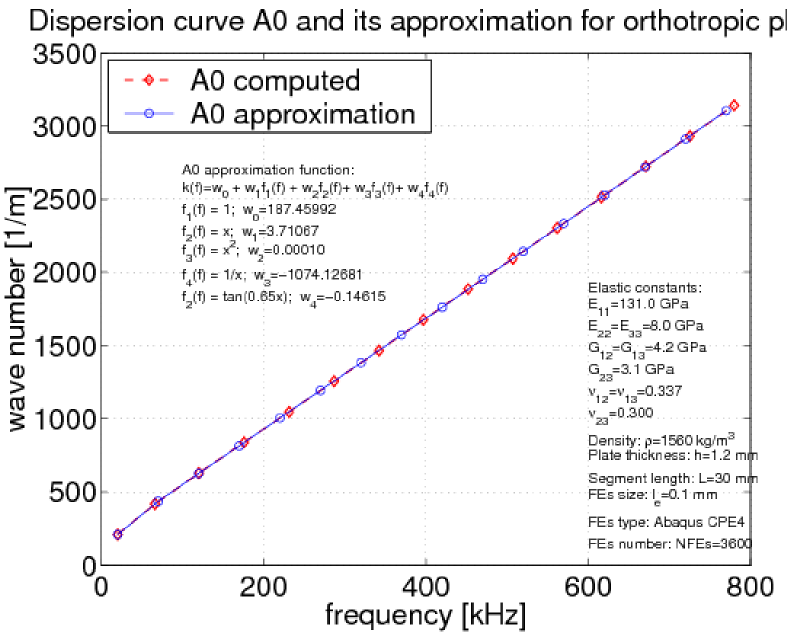

Figure 2: Fundamental dispersion curve for orthotropic plate with thickness h=1.2mm and its approximation using 5 basis functions

with mean value $\mu_0 = 131.0$ GPa and standard deviation σ_0=1.3 GPa (coefficient of variation (CoV) was 1%). Fig. 3 shows the plot of the prior (as a dashed line).

The approximate posterior distribution of Young's modulus given pseudo-experimental dispersion curves $P_N(x_k|\mathbf{y}_k)$ in the k-th step was computed using the particle filter-based identification procedure described above. In experiments, we applied $N = 2000$ particles to obtain the approximate posterior distribution and the number of steps in the sequential identification was set to K=500. Fig. 4 shows the sequential nature of the elastic constant identification process by plotting the evolution of the mean value of the posterior distribution and the corresponding plot for the one-standard deviation error bars as a function of the step number. There is also shown a solid horizontal line representing the reference Young's modulus value (131.0 GPa) applied in numerical experiments. From the plot, it may be observed that the estimation process converged to the reference value quite rapidly (in about 100 iterations).

Tab. 2 presents statistical parameters of prior and posterior distributions in the form of mean values, standard deviations and coefficients of variation (COV) are given. From the table, it can be stated that the final mean value of the posterior distribution is the same as the reference value. Moreover, the coefficient of variation decreased from 1% for the prior distribution to only 0.15% for the final posterior distribution. Fig. 3 shows the final one-dimensional posterior distribution together with the prior distribution.

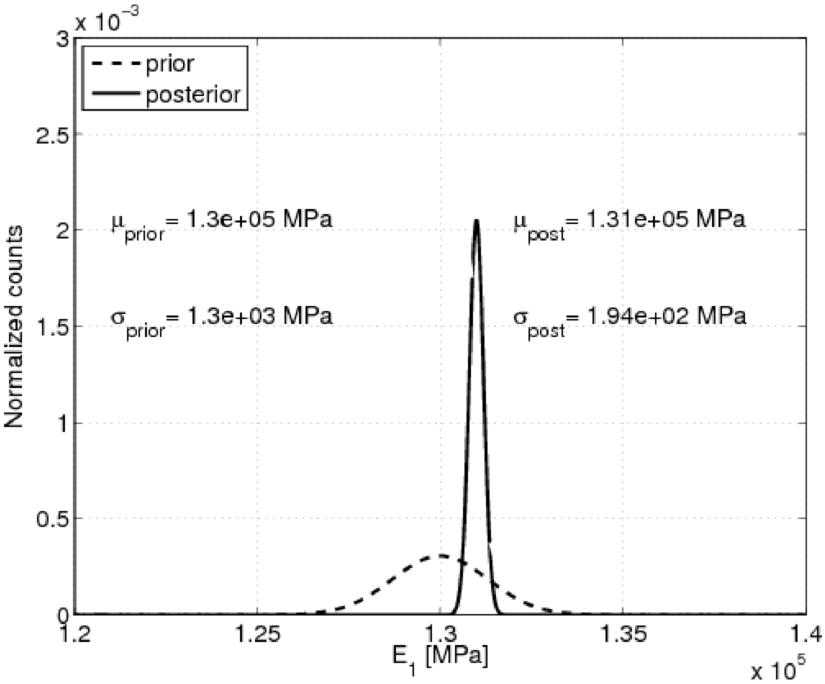

Figure 3: Prior and posterior probability density distributions for Young's modulus E_1. Prior (dashed line) has mean value $\mu_{\text{prior}} = 130.0$ GPa and standard deviation σ_{prior}=1.31 GPa (coefficient of variation (CoV) is 1%). Posterior (solid line) has mean value $\mu_{\text{post}} = 131.0$ GPa and standard deviation σ_{post}=0.19 GPa (coefficient of variation (CoV) is 0.15%)

4 FINAL CONCLUSIONS

This paper presents an application of Bayesian methods and particle filter for reconstruction of elastic parameters of plates. The proposed procedure is based on the comparison of experimental and numerical dispersion curves from guided Lamb waves monitoring.

Taking into account the assumed experimental errors and considering propagation of errors in the sequential estimation, the uncertainty in the identified value of Young's modulus E_1 is less than 0.5%. More results for other elastic parameters will be presented during the conference.

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Table 2: Statistical parameters of prior and posterior distributions for Young's modulus (mean value, standard deviation and coefficient of variation (COV))

Parameter	Prior	Posterior
Mean value (GPa)	130.0	131.0
Standard deviation (GPa)	1.3	0.19
COV (%)	1.0	0.15

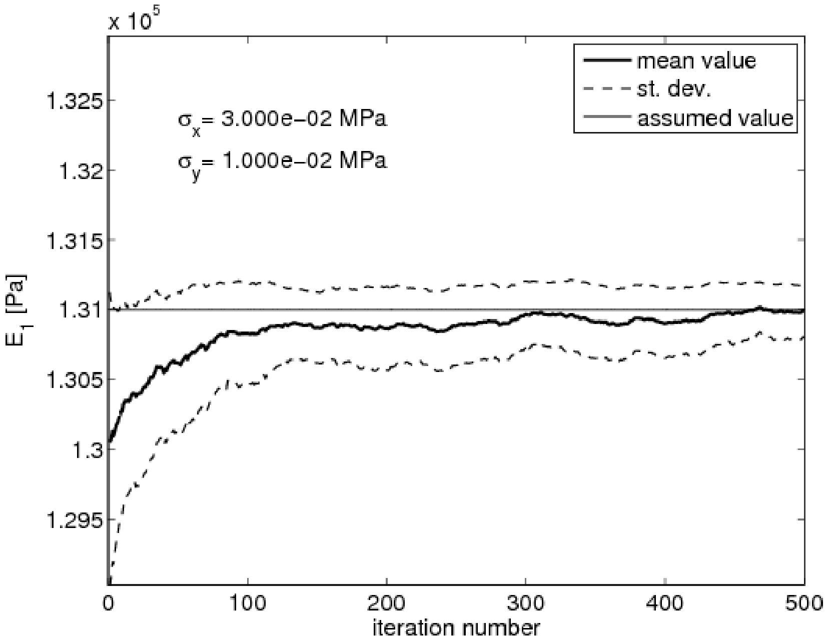

Figure 4: Plot of evolution of mean value of posterior distribution for Young's modulus and corresponding one-standard deviation error bars (solid horizontal line represents Young's modulus value (131.0 GPa) assumed in numerical experiments)

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