

Dynamic Bayesian networks for sequential parametric identification problems

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Abstract

In this work, we propose a new approach to the solution of parametric identification problems. Our approach combines measured displacements of a structure extracted from a computer vision system and computed displacements predicted by a FEM-based computer program. The parametric identification problem is solved sequentially using both sources of information about displacements and the dynamic Bayesian network with the particle filter based inference.

The proposed solution strategy is illustrated by solving material parameter identification problem of Young modulus estimation for a homogeneous isotropic laboratory-scale aluminum frame. Our approach is also verified by comparing the particle filter solution with the analytical solution obtained using the Kalman filter. The current results confirm that the proposed combination of computer vision techniques, finite element method and sequential estimation using dynamic Bayesian network can be successfully applied for solving similar parametric identification problems in mechanics of structures and materials.

Keywords: structural mechanics, parametric identification, computer vision, dynamic Bayesian network, particle filter, Kalman filter

1. Introduction

There is a growing interest in using optical measurements and on-line approach for solving parametric identification problems encountered in the context of mechanics of structures and materials. The well-known extended Kalman filter (EKF) was applied by Maier et al. [3] for stochastic estimation in fracture mechanics together with optical measurements. Furukawa and Pan [2] applied computer vision-based full-field measurements and Kalman filter for on-line characterization of anisotropic materials. Although EKF has been widely applied, it is only reliable for almost linear models. For highly-nonlinear models, particle filter may be a viable alternative.

Moreover, various current approaches to on-line structural health monitoring (SHM) relies heavily on the sequential identification of component or structure states and/or parameters for damage detection, localization and prognosis [5]. Ching et al. [1] compared the particle filter and the extended Kalman filter in the problem of Bayesian state and parameter estimation of uncertain dynamical systems. Nasrellah and Manohar [4] proposed a strategy for combining finite element method and particle filter to tackle the problem of structural system parameter identification.

This work presents an example of application of particle filter to computer vision-based on-line material parameter identification and comparison with Kalman filter.

2. Particle filter for sequential estimation

Particle filter is usually introduced in the probabilistic context for inference in dynamic Bayesian networks. Dynamic Bayesian network (DBN) is a Bayesian network which represents a temporal probability model, see [6] for a very good introduction to DBNs. The well-known Kalman filter used to model linear discrete dynamic systems, is an example of dynamic Bayesian network with continuous variables and linear Gaussian conditional distributions. On the other hand, DBN can model arbitrary dis-

tribution in which the joint distribution over the sequence of K observed variables $\mathbf{y}_{1:K}$ and state variables $\mathbf{x}_{0:K}$, is given by

$$p(\mathbf{x}_{0:K}, \mathbf{y}_{1:K}) = p(\mathbf{x}_0) \prod_{k=1}^K p(\mathbf{x}_k | \mathbf{x}_{k-1}) p(\mathbf{y}_k | \mathbf{x}_k), \quad (1)$$

where $p(\mathbf{x}_k | \mathbf{x}_{k-1})$ is the transition model (here first-order Markov chain), $p(\mathbf{y}_k | \mathbf{x}_k)$ is the observation model and $p(\mathbf{x}_0)$ is the prior distribution of initial states. Fig. 1 shows the Bayesian network structure corresponding to the first-order Markov process for the state variables and the observed variables conditioned on the state variables.

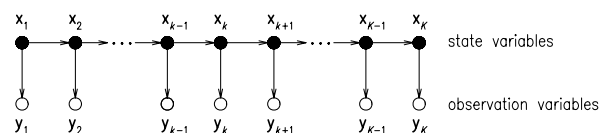


Figure 1: Dynamic Bayesian network representing the probabilistic dependence between the state and observed variables

In the sequential estimation problems, we are mainly interested in recursive computations of the posterior distribution $p(\mathbf{x}_k | \mathbf{y}_k)$ but in general exact inference is intractable so different approximate methods have been developed so far. The most successful algorithm for approximate inference is based on sequential Monte Carlo sampling and approximating the posterior using N particles to obtain the empirical distribution $P_N(\mathbf{x}_k)$.

The basic particle filter algorithm starts with a population of N initial-state samples, created by sampling from the prior $p(\mathbf{x}_0)$. Then the update cycle is repeated for each time step [6]:

1. each sample is propagated forward by sampling the next state value \mathbf{x}_k , given the current value \mathbf{x}_{k-1} for the sample, based on the transition model $p(\mathbf{x}_k | \mathbf{x}_{k-1})$.
2. each sample is weighted by the likelihood it assigns to the new evidence, $p(\mathbf{y}_k | \mathbf{x}_k)$.

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- the population is resampled to generate a new population of N samples. Each new sample is selected from the current population; the probability that a particular sample is selected is proportional to its weight.

3. Results for the example application

In this section we show how our approach can be applied to the problem of Young modulus identification of aluminum frame. The frame is shown on the right-hand side of Fig. 2 together with the main elements of the developed by the second author computer vision system. The vision system is able to track in real time the positions of markers, during a quasi-static loading of the frame. It uses digital image correlation with the accuracy of 0.15mm. More information on this system can be found in [7].

On the left-hand side of Fig. 2, FEM model of the frame and the positions of the loads are presented. The loads are applied and measured in real time by the digital force gauge Lutron FG-5000A. The bottom part of Fig. 2 presents the comparison of the displacements measured by the vision system and predicted by FEM-based computer program. From these two plots it can be concluded that the measured and predicted displacements are determined with sufficient accuracy.

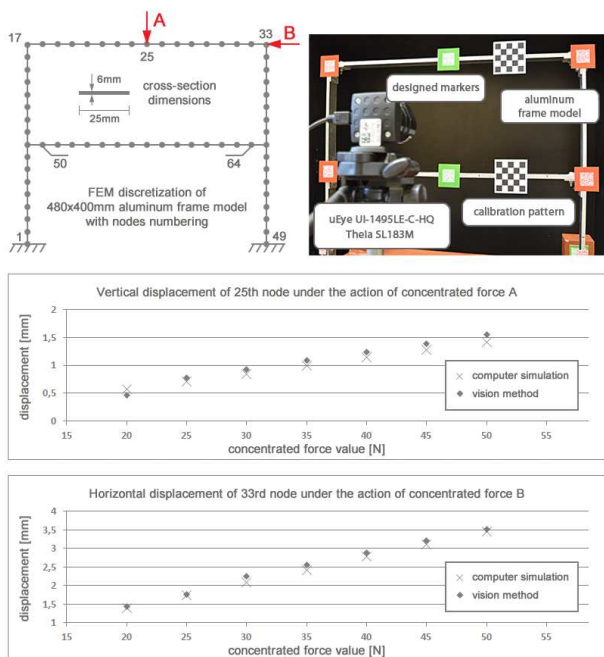


Figure 2: FEM model of the frame, test stand and plots of measured and computed displacements for two load cases.

Having the two sources of information about the frame displacements on-line, it is possible to solve the material parameter identification problem using filtering. The applied filters have only one state variable x_k which represents Young modulus in time step k . The observed variables y_k are the measured displacements. For both filters we assume the transition model for evolution of Young modulus to be static i.e. $x_k = x_{k-1}$. The prior distribution for the initial state is a normal density distribution $p(x_0) = \mathcal{N}(x_0|70GPa, 7GPa^2)$. The observation model is represented by a linear Gaussian conditional distribution with a mean vector defined by the FEM-based predicted displacements.

The presented results were obtained for particle filter with $N = 400$ particles representing Young modulus posterior distri-

bution $P_N(x_k)$. Fig. 3 shows evolution of mean values of posterior distributions for Young modulus using Kalman filter and particle filter in case of vertical force. After some number of numerical experiments, the estimated values of Young modulus by both filters have had almost the same values of 67.6 GPa and. The experiments have also shown that the influence of the prior density distribution on the final Young modulus estimates is negligibly small.

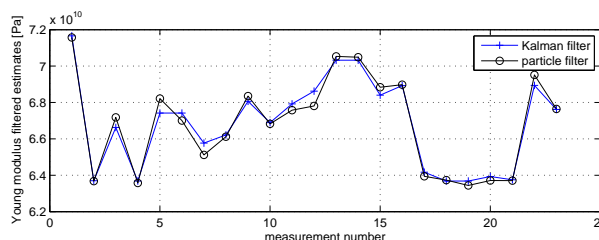


Figure 3: Plot of sequential estimation of Young modulus using Kalman and particle filters, in case of vertical force

4. Concluding remarks

In this work we have shown that the proposed sequential approach to parametric identification problems using particle filter together with computer vision based displacements measurements and FEM-based predicted displacements has been able to successfully solve the material parameter identification problem of Young modulus estimation for the aluminum laboratory-scale frame. Moreover the comparison of PF-based identification result with Kalman filter solution verified successfully our implementation of particle filter. It will allows us to test our approach for solving nonlinear identification problems.

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